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Improved Frequency Response Model Identification Method for Processes with Initial Cyclic-Steady-State

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A new nonparametric process identification method is proposed to obtain the frequency response model from given process input and output data. The proposed algorithm can estimate exact models for all desired frequencies. It is applicable to various process conditions (initial/final steady-state, initial steady-state/final cyclic-steady-state, and initial/final cyclic-steady-state) and requires a smaller amount of memory than previous methods. Also, it provides the exact models even in the presence of a static disturbance and shows an acceptable robustness to measurement noises. *AIChE J,* 57: 3429–3435, 2011
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Introduction

The describing function analysis has been widely used to identify the ultimate information of the process from the relay feedback signal since Åström and Hägglund proposed the original relay feedback identification method for the automatic tuning of proportional-integral-derivative (PID) controllers.[1] It is derived from the Fourier series of the relay feedback signal, where the fundamental term of the series is considered. In general, the ultimate frequency and gain estimated by the describing function analysis show acceptable accuracy for usual processes. However, since the square signal is approximated by one sinusoidal signal, the harmonic terms could be dominant. Several methods have been proposed to obtain a more accurate ultimate data set by reducing the harmonics.[2–4] Nevertheless, the estimation errors still remain because of the describing function approximation.

To overcome the problems of the describing function analysis method, Sung and Lee[5] proposed the Fourier analysis method. It has the ability to estimate the exact frequency response data of the process without any approximation. But, all the aforementioned methods can identify only one or two frequency response data because it uses only the cyclic-steady-state part of the process data, not sufficient to tune controller with high performance specifications. So, many researchers have exerted their efforts on developing a new algorithm to identify multiple points of frequency responses of the process.

Luyben[6] proposed an identification method for the case of initial/final steady-state using the Fourier transform. Although it can provide exact frequency models for a wide

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Table 1. Summary of Previous Approaches

Specifications	Algorithms				
	DFA[1]	FA[5]	FTS[6]	MFT[7,8]	MFTIC[10]
Applicability to initial steady-state and final steady-state	X	X	O	X	O
Applicability to initial steady-state and final cyclic-steady-state	O	O	X	O	O
Applicability to initial cyclic-steady-state and final cyclic-steady-state	O	O	X	X	O
The number of estimated frequency responses	Only one or two	Only one or two	Theoretically all	Theoretically all	Theoretically all
Accuracy	Approximated	Exact	Exact	Exact	Exact
Remarks	–	–	–	–	Needs data preprocessing step

DFA, describing function analysis; FA, Fourier analysis; FTS, Fourier transform for initial and final steady-state; MFT, modified Fourier transform for initial steady-state and final cyclic-steady-state; MTFIC, modified Fourier transform for initial cyclic-steady-state.

range of frequencies, it is valid only for the case that both the initial and the final part of the excited process data are steady-state. Sung and Lee[7] and Ma and Zhu[8] proposed an improved nonparametric identification algorithm using a modified Fourier transform. The algorithm uses all of the process data from the initial transient region to the final cyclic-steady-state part. So, it is possible for the algorithm to estimate the frequency responses of the process for all desired frequencies.[9] However, these methods can be applied only to the cases where the initial part and the final part of the excited process data are steady-state and cyclic-steady-state, respectively. They cannot incorporate the case in which both the initial part and the final part are cyclic-steady-state. So, Cheon et al.[10] proposed a new method which can be applied to a greater variety of situations (initial/final steady-state, initial steady-state/final cyclic-steady-state, and initial/final cyclic-steady-state). It also provides exact frequency responses for all desired frequencies. But, it requires a significant amount of memory because it needs a data preprocessing step of repeating the data of one period in the initial cyclic-steady-state part. Table 1 summarizes the characteristics of all the previous approaches.

Therefore, in this article, an improved frequency response model identification method is proposed to overcome the problems of the previous approaches. It has the ability to provide exact frequency responses for all desired frequencies as well as it is applicable to various conditions of processes (both initial/final steady-state, initial steady-state/final cyclic-steady-state and both initial/final cyclic-steady-state). Also, it requires a smaller amount of memory than the previous method[10] because it does not require any data preprocessing step. Furthermore, it provides the exact models even in the presence of a static disturbance and shows an acceptable robustness to measurement noises.

Process excitation

In this article, the following three types of process excitation are considered for the development of the proposed method as shown in Figure 1. In Figure 1a, both the initial and final parts are steady-state. In this case, a proportional controller is used. In Figure 1b, the initial part of the signal is steady-state and the final part is cyclic-steady-state. A biased-relay feedback method is used for this case. In Figure 1c, the

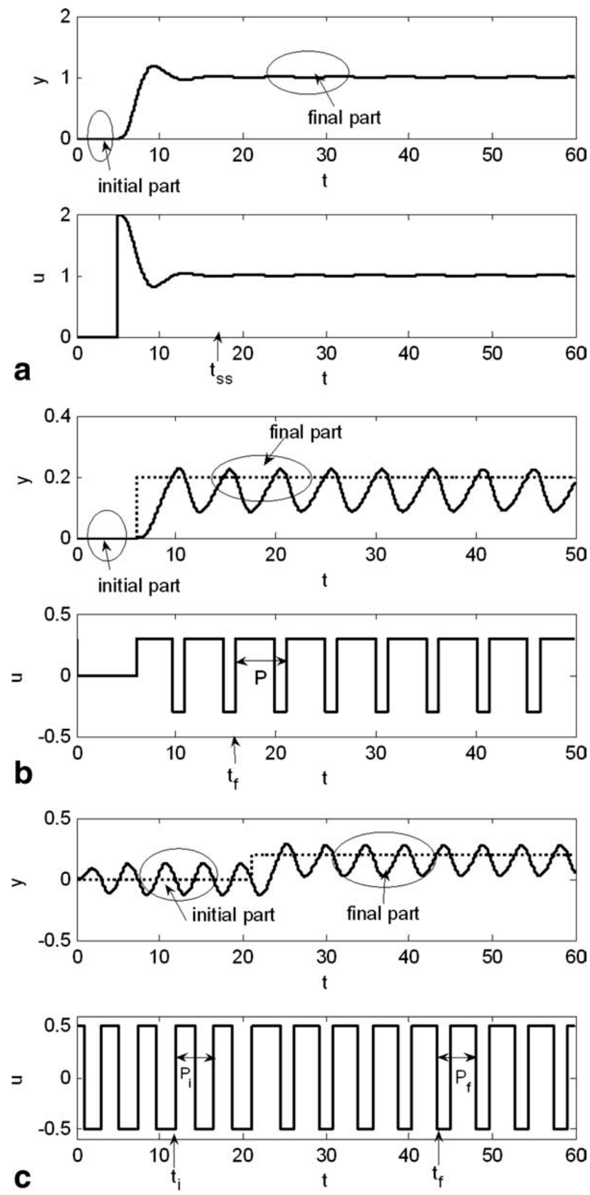

Figure 1. Three cases of process excitation.

initial unsteady-state response is stabilized to the initial cyclic-steady-state. The final part is cyclic-steady-state. In this case, a biased-relay feedback method[9] is used.

Development of Proposed Identification Method

In this section, the improved frequency response model identification method is first developed for the process input and output data of Figure 1c, followed by the extension to the other cases of Figures 1a,b.

The following assumptions and definition are considered to develop the proposed method.

Assumption 1

The initial part from t_i to $t_i + P_i$ and the final part from t_f to $t_f + P_f$ of the excited process input and output are cyclic-steady-state as shown in Figure 1c.

Assumption 2

The dynamics of the process is described by the following linear time-invariant transfer function:

$$G(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + 1} \tag{1}$$

This is equivalent to the following differential equation:

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + \cdots + a_1 \frac{dy(t)}{at} + y(t) = b_m \frac{d^m u(t)}{dt^m} + b_{m-1}\frac{d^{m-1}u(t)}{dt^{m-1}} + \cdots + b_1\frac{du(t)}{dt} + b_0 u(t), \tag{2}$$

where, u and y denote the process input and the process output, respectively.

Definition 1

Let $y_t\,(p,q,s)$ be defined as a transform of $y(\tau)$ as following:

$$y_t(p,q,s) = \int_p^q e^{-s\tau} y(\tau)d\tau, \tag{3}$$

where s is a complex variable.

Now, let us propose new algorithms to estimate the frequency responses of the process from the process input and output data and prove them.

Proposed algorithm 1

All the frequency responses of the process for any desired frequency can be estimated by the following proposed method.

For the nonzero desired frequency of $\omega \neq 0$:

$$G(j\omega) = \frac{e^{j\omega(P_i+P_f)}\int_{t_i+P_i}^{t_f+P_f} e^{-j\omega\tau}y(\tau)d\tau - e^{j\omega P_i}\int_{t_i+P_i}^{t_f} e^{-j\omega\tau}y(\tau)d\tau - e^{j\omega P_f}\int_{t_i}^{t_f+P_f} e^{-j\omega\tau}y(\tau)d\tau + \int_{t_i}^{t_f} e^{-j\omega\tau}y(\tau)d\tau}{e^{j\omega(P_i+P_f)}\int_{t_i+P_i}^{t_f+P_f} e^{-j\omega\tau}u(\tau)d\tau - e^{j\omega P_i}\int_{t_i+P_i}^{t_f} e^{-j\omega\tau}u(\tau)d\tau - e^{j\omega P_f}\int_{t_i}^{t_f+P_f} e^{-j\omega\tau}u(\tau)d\tau + \int_{t_i}^{t_f} e^{-j\omega\tau}u(\tau)d\tau} \tag{4}$$

For the zero desired frequency:

$$G(0) = \frac{p_i\int_{t_f}^{t_f+p_f} y(\tau)d\tau - p_f\int_{t_i}^{t_i+p_i} y(\tau)d\tau}{p_i\int_{t_f}^{t_f+p_f} u(\tau)d\tau - p_f\int_{t_i}^{t_i+p_i} u(\tau)d\tau} \tag{5}$$

Proof. First, several characteristics of periodic functions are needed to be considered. $d^{k-1}\,y(t)/dt^{k-1}|_{t_i} + P_i$ and $d^{k-1}\,y(t)/dt^{k-1}|_{t_i}$ are the same because $y(t)$ from t_i to $t_i + P_i$ is a periodic function of which the period is P_i. It is also valid for $u(t)$ and the final cyclic-steady-state part. Then, the following equations are obtained.

$$\int_{t_i}^{t_i+p_i}\left(d^k y(\tau)/d\tau^k\right)d\tau = d^{k-1}y(t)/dt^{k-1}\big|_{t_i+p_i} - d^{k-1}y(t)/dt^{k-1}\big|_{t_i} = 0, \quad k = 1,2,\ldots \tag{6}$$

$$\int_{t_i}^{t_i+p_i}\left(d^k u(\tau)/d\tau^k\right)d\tau = d^{k-1}u(t)/dt^{k-1}\big|_{t_i+p_i} - d^{k-1}u(t)/dt^{k-1}\big|_{t_i} = 0, \quad k = 1,2,\ldots \tag{7}$$

$$\int_{t_f}^{t_f+p_f}\left(d^k y(\tau)/d\tau^k\right)d\tau = d^{k-1}y(t)/dt^{k-1}\big|_{t_f+p_f} - d^{k-1}y(t)/dt^{k-1}\big|_{t_f} = 0, \quad k = 1,2,\ldots \tag{8}$$

$$\int_{t_f}^{t_f+p_f}\left(d^k u(\tau)/d\tau^k\right)d\tau = d^{k-1}u(t)/dt^{k-1}\big|_{t_f+p_f} - d^{k-1}u(t)/dt^{k-1}\big|_{t_f} = 0, \quad k = 1,2,\ldots \tag{9}$$

Equations 10 and 11 are obtained by integrating Eq. 2 from t_i to $t_i + P_i$ or t_f to $t_f + P_f$ and using Eqs. 6–9.

$$\int_{t_i}^{t_i+p_i} y(\tau)d\tau = b_0\int_{t_i}^{t_i+p_i} u(\tau)d\tau \tag{10}$$

$$\int_{t_f}^{t_f+p_f} y(\tau)d\tau = b_0\int_{t_f}^{t_f+p_f} u(\tau)d\tau \tag{11}$$

Now, consider the following property of the transform, derived easily by integration by parts.

$$y_t^{(n)}(p,q,s) = e^{-sq}y^{(n-1)}(q) - e^{-sp}y^{(n-1)}(p) + se^{-sq}y^{(n-2)}(q) - se^{-sp}y^{(n-2)}(p) + \cdots + s^{n-2}e^{-sq}y^{(1)}(q) - s^{n-2}e^{-sp}y^{(1)}(p) + s^{n-1}e^{-sq}y(q) - s^{n-1}e^{-sp}y(p) + s^n\int_p^q e^{-s\tau}y(\tau)d\tau \tag{12}$$

where $y^{(i)} = d^i\,y(t)/dt^i$ and $y_t^{(n)}(p,q,s) = \int_p^q e^{-s\tau}\{d^n y(\tau)/d\tau^n\}d\tau$.

By applying the transformation of Eq. 3 to Eq. 2 and using Eq. 12, the following is obtained.

$$
\begin{aligned}
&a_n\left\{e^{-sq}y^{(n-1)}(q)-e^{-sp}y^{(n-1)}(p)+\cdots+s^{n-1}e^{-sq}y(q)-s^{n-1}e^{-sp}y(p)+s^n\int_p^q e^{-s\tau}y(\tau)d\tau\right\}+\cdots\\
&+a_2\left\{e^{-sq}y^{(1)}(q)-e^{-sp}y^{(1)}(p)+se^{-sq}y(q)-se^{-sp}y(p)+s^2\int_p^q e^{-s\tau}y(\tau)d\tau\right\}+a_1\left\{e^{-sq}y(q)-e^{-sp}y(p)+s\int_p^q e^{-s\tau}y(\tau)d\tau\right\}\\
&+\int_p^q e^{-s\tau}y(\tau)d\tau=b_m\left\{e^{-sq}u^{(m-1)}(q)-e^{-sp}u^{(m-1)}(p)+\cdots+s^{m-1}e^{-sq}u(q)-s^{m-1}e^{-sp}u(p)+s^m\int_p^q e^{-s\tau}u(\tau)d\tau\right\}+\cdots\\
&\qquad+b_2\left\{e^{-sq}u^{(1)}(q)-e^{-sp}u^{(1)}(p)+se^{-sq}u(q)-se^{-sp}u(p)+s^2\int_p^q e^{-s\tau}u(\tau)d\tau\right\}\\
&\qquad\qquad+b_1\left\{e^{-sq}u(q)-e^{-sp}u(p)+s\int_p^q e^{-s\tau}u(\tau)d\tau\right\}+b_0\int_p^q e^{-s\tau}u(\tau)d\tau
\end{aligned}
\tag{13}
$$

Applying the following steps to both sides of Eq. 13, Eq. 14 is obtained.

Step 1: multiply e^{sq}.

Step 2: integrate with respect to q between t_f and $t_f + P_f$ and simplify the equation using Eqs. 6–9.

Step 3: multiply e^{sp}.

Step 4: integrate with respect to p between t_i and $t_i + P_i$ and simplify the equation using Eqs. 6–9.

$$
\begin{aligned}
&\mathrm{den}(s)\int_{t_i}^{t_i+P_i} e^{sp}\int_{t_f}^{t_f+P_f} e^{sq}\int_p^q e^{-s\tau}y(\tau)d\tau dq dp+\left(\frac{\mathrm{den}(s)-1}{s}\right)\\
&\times\left(\int_{t_i}^{t_i+P_i} e^{sp}dp\int_{t_f}^{t_f+P_f} y(q)dq-\int_{t_f}^{t_f+P_f} e^{sq}dq\int_{t_i}^{t_i+P_i} y(p)dp\right)\\
&=\mathrm{num}(s)\int_{t_i}^{t_i+P_i} e^{sp}\int_{t_f}^{t_f+P_f} e^{sq}\int_p^q e^{-s\tau}u(\tau)d\tau dq dp\\
&+\left(\frac{\mathrm{den}(s)-b_0}{s}\right)\left(\int_{t_i}^{t_i+P_i} e^{sp}dp\int_{t_f}^{t_f+P_f} u(q)dq\right.\\
&\left.-\int_{t_i}^{t_i+P_i} u(p)dp\int_{t_f}^{t_f+P_f} e^{sq}dq\right)
\end{aligned}
\tag{14}
$$

where $\mathrm{den}(s) = a_n\, s^n + a^{n-1}\, s^{n-1} + \cdots + a_1\, s + 1$ and $\mathrm{num}(s) = b_m\, s^m + b^{m-1}\, s^{m-1} + \cdots + b_1\, s + b_0$.

Considering Eqs. 10 and 11, Eq. 14 can be simplified to Eq. 15.

$$
G(s)=\frac{num(s)}{den(s)}=\frac{B+s^2\int_{t_i}^{t_i+P_i} e^{sp}\int_{t_f}^{t_f+P_f} e^{sq}\int_p^q e^{-s\tau}y(\tau)d\tau dq dp}{A+s^2\int_{t_i}^{t_i+P_i} e^{sp}\int_{t_f}^{t_f+P_f} e^{sq}\int_p^q e^{-s\tau}u(\tau)d\tau dq dp}
\tag{15}
$$

where

$$
A=\int_{t_f}^{t_f+P_f} u(q)dq\left[e^{s(t_i+P_i)}-e^{st_i}\right]-\int_{t_i}^{t_i+P_i} u(p)dp\left[e^{s(t_f+P_f)}-e^{st_f}\right]
$$

and

$$
B=\int_{t_f}^{t_f+P_f} y(q)dq\left[e^{s(t_i+P_i)}-e^{st_i}\right]-\int_{t_i}^{t_i+P_i} y(p)dp\left[e^{s(t_f+P_f)}-e^{st_f}\right]
$$

Equation 16 is obtained by integration by parts as follows:

$$
G(s)=\frac{e^{s(P_i+P_f)}\int_{t_i+P_i}^{t_f+P_f} e^{-s\tau}y(\tau)d\tau-e^{sP_i}\int_{t_i+P_i}^{t_f} e^{-s\tau}y(\tau)d\tau-e^{sP_f}\int_{t_i}^{t_f+P_f} e^{-s\tau}y(\tau)d\tau+\int_{t_i}^{t_f} e^{-s\tau}y(\tau)d\tau}{e^{s(P_i+P_f)}\int_{t_i+P_i}^{t_f+P_f} e^{-s\tau}u(\tau)d\tau-e^{sP_i}\int_{t_i+P_i}^{t_f} e^{-s\tau}u(\tau)d\tau-e^{sP_f}\int_{t_i}^{t_f+P_f} e^{-s\tau}u(\tau)d\tau+\int_{t_i}^{t_f} e^{-s\tau}u(\tau)d\tau}
\tag{16}
$$

Substituting $j\omega$ for s gives the proposed algorithm 1 of Eq. 4. Equation 5 for the zero frequency is obtained by applying L'Hospital's rule to Eq. 4. ■

Now, all the frequency response data of the process for any desired frequency points can be estimated by the proposed method 1 of Eqs. 4 and 5 repetitively for each frequency provided that the excited process signal contains the corresponding frequency component. The proposed algorithm 1 provides exact frequency data sets for all desired frequencies because it is developed without any approximation. Also, it need not have to store the process input and output data. Meanwhile, the most advanced previous method requires a significant amount of memory for the data preprocessing step.[10]

Extensions to the Other Cases

The proposed algorithm 1 for processes with both initial and final cyclic-steady-state can be extended to the other cases of Figures 1a,b. Figures 1a,b represent the cases of initial/final steady-state and initial steady-state/final cyclic-steady-state, respectively.

Proposed Algorithm 2 for initial and final steady-state

The frequency responses of the process for all desired frequencies can be estimated by Eq. 17 for the case that both the initial part and the final part are steady-state.

$$G(j\omega)=\frac{-\omega\int_{t_i}^{t_f}e^{-j\omega\tau}y(\tau)d\tau+je^{-j\omega t_f}y(t_f)-je^{-j\omega t_i}y(t_i)}{-\omega\int_{t_i}^{t_f}e^{-j\omega\tau}u(\tau)d\tau+je^{-j\omega t_f}u(t_f)-je^{-j\omega t_i}u(t_i)} \quad (17)$$

Proof. Note that Figure 1a is the case of Figure 1c with $P_i = P_f = \mathrm{P} \to 0$. Then, Eq. 4 with $P_i = P_f = \mathrm{P} \to 0$ can be applied to Figure 1a as follows:

$$G(s)=\lim_{P\to 0}\frac{e^{2sP}\int_{t_i+P}^{t_f+P}e^{-s\tau}y(\tau)d\tau-e^{sP}\int_{t_i+P}^{t_f}e^{-s\tau}y(\tau)d\tau-e^{sP}\int_{t_i}^{t_f+P}e^{-s\tau}y(\tau)d\tau+\int_{t_i}^{t_f}e^{-s\tau}y(\tau)d\tau}{e^{2sP}\int_{t_i+P}^{t_f+P}e^{-s\tau}u(\tau)d\tau-e^{sP}\int_{t_i+P}^{t_f}e^{-s\tau}u(\tau)d\tau-e^{sP}\int_{t_i}^{t_f+P}e^{-s\tau}u(\tau)d\tau+\int_{t_i}^{t_f}e^{-s\tau}u(\tau)d\tau} \quad (18)$$

By L'Hospital's rule, Eq. 18 can be simplified to Eq. 19.

$$G(s)=\frac{s^2\int_{t_i}^{t_f}e^{-s\tau}y(\tau)d\tau+se^{-st_f}y(t_f)-se^{-st_i}y(t_i)}{s^2\int_{t_i}^{t_f}e^{-s\tau}u(\tau)d\tau+se^{-st_f}u(t_f)-se^{-st_i}u(t_i)} \quad (19)$$

Then, it is straightforward to obtain Eq. 17 from Eq. 19 by substituting $j\omega$ for s. ■

Proposed Algorithm 3 for initial steady-state and final cyclic steady-state

The frequency responses of the process for all desired frequencies can be estimated by (20)-(21) for the case that the initial part is steady-state and the final part is cyclic-steady-state.

For the nonzero desired frequency of $\omega \neq 0$:

$$G(j\omega)=\frac{(e^{j\omega P_f}-1)\left(j\omega\int_{t_i}^{t_f}e^{-j\omega\tau}y(\tau)d\tau-e^{-j\omega t_i}y(t_i)\right)+j\omega e^{j\omega P_f}\int_{t_f}^{t_f+P_f}e^{-j\omega\tau}y(\tau)d\tau}{(e^{j\omega P_f}-1)\left(j\omega\int_{t_i}^{t_f}e^{-j\omega\tau}u(\tau)d\tau-e^{-j\omega t_i}u(t_i)\right)+j\omega e^{j\omega P_f}\int_{t_f}^{t_f+P_f}e^{-j\omega\tau}u(\tau)d\tau} \quad (20)$$

For the zero frequency:

$$G(0)=\frac{-P_f y(t_i)+\int_{t_f}^{t_f+P_f}y(\tau)d\tau}{-P_f u(t_i)+\int_{t_f}^{t_f+P_f}u(\tau)d\tau} \quad (21)$$

Proof. Note that Figure 1b is the case of Figure 1c with $P_i \to 0$. Then, Eq. 4 with $P_i \to 0$ can be applied to Figure 1b. By applying L'Hospital's rule to Eq. 4, Eq. 22 can be obtained.

$$G(s)=\frac{(e^{sP_f}-1)\left(s\int_{t_i}^{t_f}e^{-s\tau}y(\tau)d\tau-e^{-st_i}y(t_i)\right)+se^{sP_f}\int_{t_f}^{t_f+P_f}e^{-s\tau}y(\tau)d\tau}{(e^{sP_f}-1)\left(s\int_{t_i}^{t_f}e^{-s\tau}u(\tau)d\tau-e^{-st_i}u(t_i)\right)+se^{sP_f}\int_{t_f}^{t_f+P_f}e^{-s\tau}u(\tau)d\tau} \quad (22)$$

Then, it is straightforward to obtain Eq. 20 from Eq. 22 by substituting $j\omega$ for s. Equation 21 can be derived by applying L'Hospital's rule to Eq. 20. ■

Now, it is clear that the proposed methods of Eqs. 4, 5, 17, 20, and 21 can provide exact frequency responses of the process for various situations of Figure 1 (initial and final steady-state, initial steady-state/final cyclic-steady-state, initial and final cyclic-steady-state). Also, it can estimate all the frequency response models for any desired frequency without any modeling errors.

Discussions on Effects of Static Disturbances

Assume that a static disturbance d_{in} is added to the process input. The proposed algorithm is not affected by the disturbance and provides the same estimates as those of the case with no static disturbance.

Proof. If the static input disturbance d_{in} is added to the process input, then the denominator of the algorithm of Eq. 4 becomes

$$\begin{aligned}
&e^{j\omega(P_i+P_f)}\int_{t_i+P_i}^{t_f+P_f}e^{-j\omega\tau}(u(\tau)+d_{in})d\tau-e^{j\omega P_i}\int_{t_i+P_i}^{t_f}e^{-j\omega\tau}(u(\tau)+d_{in})d\tau\\
&\quad-e^{j\omega P_f}\int_{t_i}^{t_f+P_f}e^{-j\omega\tau}(u(\tau)+d_{in})d\tau+\int_{t_i}^{t_f}e^{-j\omega\tau}(u(\tau)+d_{in})d\tau\\
&=e^{j\omega(P_i+P_f)}\int_{t_i+P_i}^{t_f+P_f}e^{-j\omega\tau}u(\tau)d\tau-e^{j\omega P_i}\int_{t_i+P_i}^{t_f}e^{-j\omega\tau}u(\tau)d\tau\\
&\quad-e^{j\omega P_f}\int_{t_i}^{t_f+P_f}e^{-j\omega\tau}u(\tau)d\tau+\int_{t_i}^{t_f}e^{-j\omega\tau}u(\tau)d\tau\\
&\quad+d_{in}\left\{e^{j\omega(P_i+P_f)}\int_{t_i+P_i}^{t_f+P_f}e^{-j\omega\tau}d\tau-e^{j\omega P_i}\int_{t_i+P_i}^{t_f}e^{-j\omega\tau}d\tau\right.\\
&\quad\left.-e^{j\omega P_f}\int_{t_i}^{t_f+P_f}e^{-j\omega\tau}d\tau+\int_{t_i}^{t_f}e^{-j\omega\tau}d\tau\right\}\\
&=e^{j\omega(P_i+P_f)}\int_{t_i+P_i}^{t_f+P_f}e^{-j\omega\tau}u(\tau)d\tau-e^{j\omega P_i}\int_{t_i+P_i}^{t_f}e^{-j\omega\tau}u(\tau)d\tau\\
&\qquad-e^{j\omega P_f}\int_{t_i}^{t_f+P_f}e^{-j\omega\tau}u(\tau)d\tau+\int_{t_i}^{t_f}e^{-j\omega\tau}u(\tau)d\tau \quad (23)
\end{aligned}$$

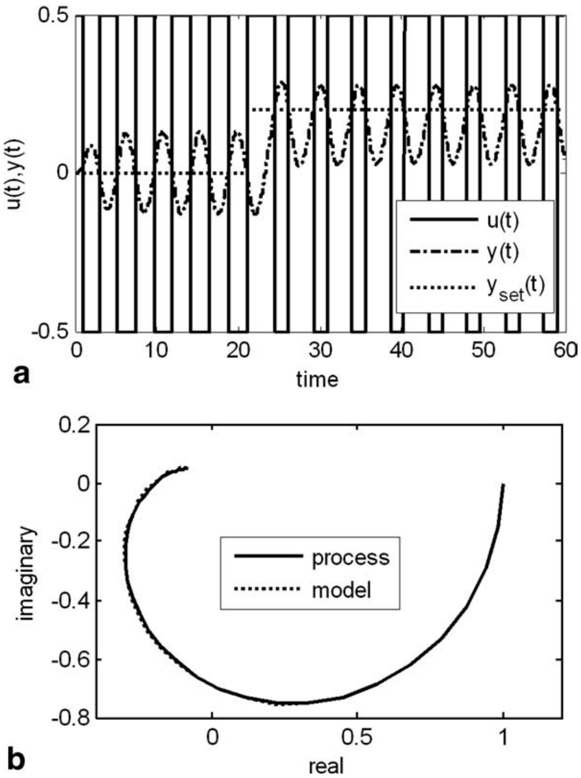

Figure 2. (a) Process excitation (initial cyclic-steady-state and final cyclic-steady-state) and (b) identified frequency responses.

Equation 23 is easily derived by replacing $\int_{t_1}^{t_2} e^{-j\omega\tau} d\tau$ by $(e^{-j\omega t_2} - e^{-j\omega t_1})/(-j\omega)$. ■

Equation 23 means that the proposed algorithm is not affected by the static disturbances at all. That is, the proposed algorithm provides the same estimates as those of the case with no disturbance. Note that wrong deviation variables are equivalent to the case of static disturbance. It means that the proposed method provides the exact estimates even in the presence of wrong deviation variables.

In summary, the proposed frequency response model identification method has several remarkable advantages as follows: First, the proposed algorithm provides the exact frequency response data for all the desired frequencies. Second, it can be applied to the various conditions of process (initial and final steady-state, initial steady-state/final cyclic-steady-state, and initial and final cyclic-steady-state). Third, it does not require a data memory to store the process input and output data. Fourth, it estimates exact frequency responses data even under the environment of static disturbances. Fifth, the proposed algorithm provides the exact estimates even in the presence of wrong deviation variables.

Simulation Study

The performances of the proposed algorithm are validated with several simulation studies. First, the accuracy of the estimated frequency responses and applicability to various process conditions are confirmed. Next, the effect of disturbance and measurement noise on the performance of the proposed algorithm is validated.

Consider the following third-order plus time delay process:

$$G(s) = \frac{e^{-0.2s}}{(s+1)^3} \tag{24}$$

Figure 2 shows the excitation of the process (Eq. 24) by the relay feedback method and the estimation accuracy of the proposed algorithms of Eqs. 4 and 5 for the case of both initial and final cyclic-steady-state. As expected, the proposed method provides exact frequency responses for all the desired frequencies. The other cases (both initial and final steady-state and initial steady-state/final cyclic-steady-state) were also simulated and the ability of the proposed algorithms of Eq. 17 and Eqs. 20 and 21 to estimate the exact frequency response data of the process is confirmed.

Figure 3a shows the input/output data of the process (Eq. 24) in the presence of an input static disturbance of 0.1. Figure 3b compares the Nyquist plots of the process and the model. As expected, the proposed algorithm completely removes the effect of the static input disturbance and provides the exact frequency responses of the process.

Figure 4a shows the input/output data of the process (Eq. 24) of which the output is corrupted by random measurement noises distributed uniformly between −0.02 and 0.02. Figure 4b shows that the proposed algorithm provides acceptable accuracy and robustness to measurement noises.

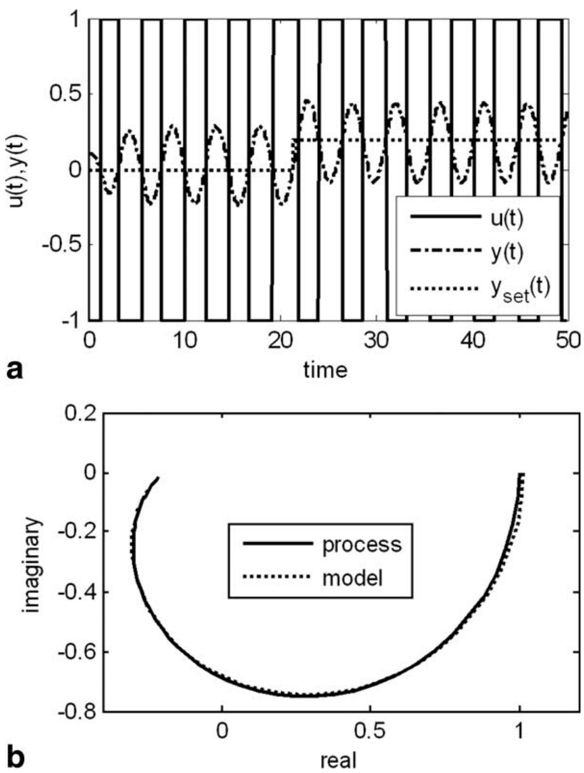

Figure 3. (a) Process excitation in the case of a static input disturbance and (b) identified frequency responses.

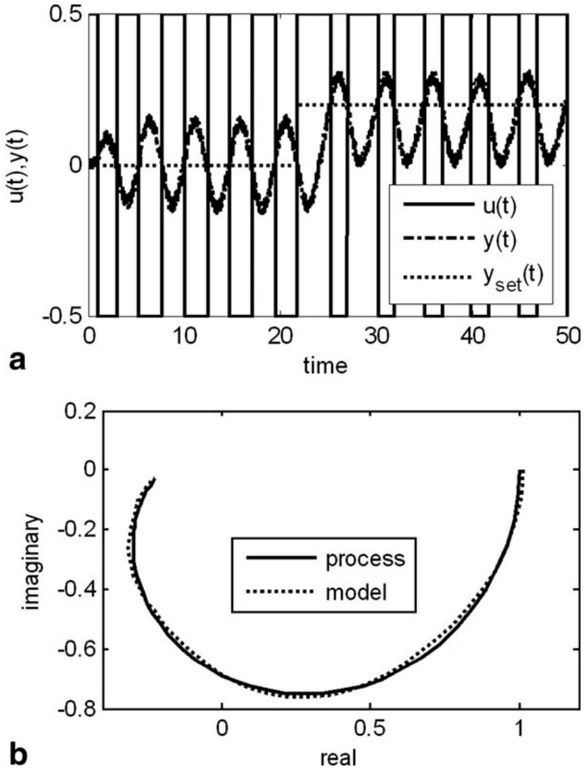

Figure 4. (a) Process excitation in the case of measurement noises and (b) identified frequency responses.

Conclusions

In this study, a new frequency response model identification method is proposed to overcome the problems of the previous approaches. The proposed algorithm provides accurate frequency responses data sets for all desired frequencies and can be applied to various process conditions. It need not have to store the process input and output data, while the most advanced previous method[10] requires a significant amount of memory for a data preprocessing step. Furthermore, the proposed method completely removes the effects of static disturbances and shows good robustness to measurement noises.

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